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# A collective variable approach and stabilization for dispersion-managed optical solitons in the quintic complex Ginzburg–Landau equation as perturbations of the nonlinear Schrödinger equation

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**Abstract**

With the help of the one-dimensional quintic complex Ginzburg–Landau equation (CGLE) as perturbations of the nonlinear Schrödinger equation (NLSE), we derive the equations of motion of pulse parameters called collective variables (CVs), of a pulse propagating in dispersion-managed (DM) fibre optic links. The equations obtained are investigated numerically in order to view the evolution of pulse parameters along the propagation distance, and also to analyse effects of initial amplitude and width on the propagating pulse. Nonlinear gain is shown to be beneficial in stabilizing DM solitons. A fully numerical simulation of the one-dimensional quintic CGLE as perturbations of NLSE finally tests the results of the CV theory. A good agreement is observed between both methods.

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## 1. Introduction

The concept of solitons describes various physical phenomena ranging from solitary waves on a water surface to ultrashort optical pulses from a laser. The study of optical solitons is interesting for its fundamental aspect as well as for its important applications [1, 2]. Optical solitons may soon be the primary carriers for long- and short-distance information transmission because, unlike pulses in a linear dispersive fibre, solitons are self-confined, propagating for a long distance without changing shape. A well-known example of an equation which admits pulse-like soliton solutions is the NLSE [1, 2]. It has rich properties of Hamiltonian systems, i.e. conservative systems. Up to now, the NLSE still attracts a lot of attention from the scientific community due to the fact that it describes the propagation of a pulse in a nonlinear Kerr medium. Optical solitons propagating in optical fibres may induce a host of nonlinear phenomena such as parametric wave mixing, stimulated Raman scattering, or self-steepening [1–4].

For long-distance communication systems, compensating for attenuation of pulses inherent in the fibre, is an important issue. One approach is the use of periodically spaced phase-sensitive amplifiers. Each such amplifier exhibits an associated reference phase. The part of the signal in phase with this reference phase is amplified, while the out-of-phase component is attenuated [5–7]. In the second approach, the losses can be compensated by the erbium-doped amplifiers [2]. A well-known model for the study of pulse propagation in doped fibre amplifiers is the one-dimensional quintic CGLE in a dimensionless form

$$\begin{aligned} &\mathrm{i}\psi_z + (p_r(z) + \mathrm{i}p_i(z))\psi_{tt} + (q_r(z) + \mathrm{i}q_i(z))|\psi|^2\psi \\ &\qquad + (c_r(z) + \mathrm{i}c_i(z))|\psi|^4\psi = \mathrm{i}(\gamma_r(z) + \mathrm{i}\gamma_i(z))\psi, \end{aligned} \tag{1}$$

where $\psi(z, t)$ is the envelope amplitude of the electric field, $t$ is the retarded time and $z$ is the propagation distance. The parameters $p_r$, $p_i$, $q_r$, $q_i$, $c_r$, $c_i$, $\gamma_r$ and $\gamma_i$ are real constants. $p_r$ measures the wave dispersion, $p_i$ the spectral filtering, $q_r$ and $q_i$ represent the nonlinear coefficient and the nonlinear gain-absorption coefficient, respectively. We noted that nonlinear gain helps to suppress the growth of radiative background (linear mode) which always accompanies the propagation of nonlinear stationary pulses in real fibre links. $c_r$ and $c_i$ stand for the higher-order correction terms to the nonlinear refractive index and the nonlinear amplification absorption, respectively. $\gamma_r$ and $\gamma_i$ represent the linear gain and the frequency shift, respectively.

The quintic CGLE plays an important role in many branches of physics, including binary fluid convection, phase transition and many phenomena in optics where it is often used to model several types of passively mode-locked lasers with saturable absorbers, parametric oscillators, transverse soliton effects in wide aperture lasers and wave propagation in nonlinear optical fibres with gain and spectral filtering [8–10]. The quintic CGLE has also been used to describe pattern formation near a hopf bifurcation and has become a paradigmatic model for the study of spatiotemporal chaos. The one-dimensional quintic CGLE possesses a rich variety of soliton solutions, including coherent structures such as pulses (solitary waves), fronts (shock waves), sources, sinks [11, 12] and new fascinating types of pulsating soliton. Namely, different types of localized pulsating solutions such as plain pulsating, exploding (erupting), creeping and chaotic solutions have been found [13]. Mode-locked lasers which are typically modelled using the quintic CGLE allow experimental observation of temporal soliton undergoing dramatic transients which we dub exploding soliton. During an explosion, the soliton energy and spectrum undergo dramatic changes, but return to the steady-state value afterwards [14]. The study of the effect of nonlinear gradient terms on pulsating, exploding and creeping solitons has been made [15]. The obtained results show that the nonlinear gradient terms can change both the pulsating and the erupting solitons into fixed shape solitons, which are meaningful for practical use such as to realize experimentally the undistorded transmission of femtosecond pulse in optical fibres. For the creeping soliton, the nonlinear gradient terms will make the soliton breathe periodically at different frequencies on one side and rapidly spread on the other side.

One of the major lines of recent research demonstrates that DM solitons for data transmissions design will substantially increase the capacity of the fibre optic link [16]. Basically, the DM technique consists of using a transmission line with a periodic map, such that each period is built up by two types of fibre of generally different lengths and opposite group velocity dispersion [16]. Dispersion-managed solitons are attracting considerable interest in optical communication systems because of their superb characteristics which are not observed with conventional solitons. Furthermore, transmission performance are sometimes degraded by perturbations (as linear waves). Actually, the propagation of pulse in fibre links is always destabilized. The use of transmission control methods such as guiding filters [17–19] were studied in order to stabilize DM solitons propagation. In addition, it is shown that nonlinear gain is expected to be more beneficial to DM solitons than to conventional solitons [20], in order to stabilize DM soliton transmissions. Sufficiently strong periodic DM allows for stationary propagation of nonlinear return-to-zero pulses with finite energy when the average dispersion is close or even equal to zero [21]. For this reason, as can be seen in equation (1), the constant parameters $p, q, c$ and $\gamma$ are commonly expressed as functions of $z$ (without loosing their constant character), i.e. $p = p_r(z) + \mathrm{i}p_i(z)$, $q = q_r(z) + \mathrm{i}q_i(z)$, $c = c_r(z) + \mathrm{i}c_i(z)$ and $\gamma = \gamma_r(z) + \mathrm{i}\gamma_i(z)$, respectively.

Since the wording DM has been originally introduced in a transmission line modelled by the NLSE, the quintic CGLE can be rewritten as perturbations of NLSE in the following way:

$$\begin{aligned} \mathrm{i}\psi_z + p_r(z)\psi_{tt} + q_r(z)|\psi|^2\psi &= \mathrm{i}[(\gamma_r(z) + \mathrm{i}\gamma_i(z))\psi] \\ &\quad - \mathrm{i}[p_i(z)\psi_{tt} + q_i(z)|\psi|^2\psi + \mathrm{i}c_i(z))|\psi|^4\psi] + c_r(z)|\psi|^4\psi. \end{aligned} \tag{2}$$

Various analytical treatments have been proposed to describe the main characteristics of the pulse evolution [22–24]. Among these various treatments, a well-studied method is the so-called variational method involving a Gaussian trial function which provides explicit (although approximate) analytical expressions for the pulse compression/decompression factor, the maximum pulse amplitude and the induced frequency chirp [22]. The purpose and optimization of the soliton transmission systems are fundamentally based on the general particle-like nature of solitons. This particle-like behaviour has led to the formulation of collective variable (CV) techniques for DM soliton, to obtain more insight into their dynamical behaviour, since no exact analytical solution for DM soliton exists up to date [16]. Then, each degree of freedom of the soliton is associated with a variable, called a collective variable (CV), describing a physical parameter for the pulse as amplitude, chirp, frequency, pulse width, and so on [25–28]. The proposed (CV) method allows us to obtain the explicit analytical expression for the CVs equations of motion.

In this paper, we investigate effects of nonlinear gain and higher-order correction term of the nonlinear refractive index on the propagation and stabilization of DM soliton via a CV analytical approach. In section 2, we present the derivation of the CVs equations of motion. Section 3 is devoted to the results of numerical investigations and comparisons between the direct numerical solution of the one-dimensional quintic CGLE as perturbations of NLSE and the analytical results of the CV theory are made. Section 4 concludes the paper.

## 2. Derivation of the CVs equations of motion

Let us first consider a decomposition of the original field, say $\psi(z, t)$ at position $z$ in the fibre and at time $t$, as follows [27]:

$$\psi(z, t) = f(X_1, X_2, \ldots, X_N, t) + g(z, t), \tag{3}$$

where parameters of the pulse are symbolically represented by $X_j$, $j = 1, \ldots, N$ in the theoretical treatment of their dynamics; the ansatz function $f$ is chosen to be the best representation of the pulse configuration and $g(z, t)$ is the remaining field that we call residual field, accounts for the dressing of the soliton and any radiation coupled to the soliton's motion.

The substitution of equation (3) into the quintic CGLE (1) yields directly the equation of motion for the residual field as

$$g_z = \mathrm{i}p(z)g_{tt} + \mathrm{i}q(z)|f+g|^2 g + \mathrm{i}c(z)|f+g|^4 g + \gamma(z)g \\ - \sum_{j=1}^{N} \dot{X}_j f_{X_j} + \mathrm{i}p(z)f_{tt} + \mathrm{i}q(z)|f+g|^2 f + \mathrm{i}c(z)|f+g|^4 f + \gamma(z)f. \tag{4}$$

The overhead dot represents the derivative with respect to $z$ and the subscripts $X_j$ denote partial derivative. To constrain the system of new variables (CVs and $g$) to remain in the same phase space as originally (see equation (1)), it is usual to minimize the residual field energy $\epsilon$. That measures the correctness (accuracy) of the ansatz function $f$. According to the set of constraints given in [27], we have

$$-\dot{C}_j = -\langle f^\star_{X_j} g_z \rangle - \sum_{k=1}^{N} \dot{X}_k \langle f^\star_{X_j X_k} g \rangle + \text{c.c.} \tag{5}$$

where c.c. stands for the complex conjugate and $\langle \cdots \rangle$means $\int_{-\infty}^{+\infty} (\cdots)\mathrm{d}t$. Substituting $g_z$ from equation (4) into equation (5) leads to the following matrix equation:

$$-[\dot{\mathbf{C}}] = \left[\frac{\partial \mathbf{C}}{\partial \mathbf{X}}\right][\dot{\mathbf{X}}] + [\mathbf{R}] \tag{6}$$

with

$$R_k = -2\,\mathrm{Re}\langle \mathrm{i}pf^\star_{X_k} g_{tt}\rangle - 2\,\mathrm{Re}\langle \mathrm{i}qf^\star_{X_k}|f+g|^2 g\rangle - 2\,\mathrm{Re}\langle \mathrm{i}cf^\star_{X_k}|f+g|^4 g\rangle \\ -2\,\mathrm{Re}\langle \gamma f^\star_{X_k} g\rangle - 2\,\mathrm{Re}\langle \mathrm{i}pf^\star_{X_k} f_{tt}\rangle - 2\,\mathrm{Re}\langle \mathrm{i}qf^\star_{X_k}|f+g|^2 f\rangle \\ -2\,\mathrm{Re}\langle icf^\star_{X_k}|f+g|^4 f\rangle - 2\,\mathrm{Re}\langle \gamma f^\star_{X_k} f\rangle \tag{7}$$

and

$$\frac{\partial C_j}{\partial X_k} = 2\int_{-\infty}^{+\infty} \mathrm{Re}\left[f^\star_{X_j} f_{X_k}\right]\mathrm{d}t - 2\int_{-\infty}^{+\infty}\left[g f^\star_{X_j X_k}\right]\mathrm{d}t. \tag{8}$$

The final CVs equations of motion are obtained by setting the constraint term to zero in equation (6), and also by setting the residual field to zero ($g(z,t) = 0$) for the lowest-order approximation. We assume the desired form of the function $f$ as a Gaussian profile given by

$$f = X_1 \exp\left[-\frac{(t-X_2)^2}{X_3^2} + \mathrm{i}\frac{X_4}{2}(t-X_2)^2 + \mathrm{i}X_5(t-X_2) + \mathrm{i}X_6\right] \tag{9}$$

where $X_1$, $X_2$, $\sqrt{2\ln 2}X_3$, $X_4/2\pi$, $X_5/2\pi$ and $X_6$ represent the pulse amplitude, temporal position, pulse width (FWHM), chirp, frequency and phase, respectively. Hence, the following inverse matrix occurs:

$$\left[\frac{\partial \mathbf{C}}{\partial \mathbf{X}}\right]^{-1} = \frac{1}{\sqrt{2\pi}}\begin{pmatrix} \frac{3}{2X_3} & 0 & -\frac{1}{X_1} & 0 & 0 & 0 \\ 0 & \frac{X_3}{X_1^2} & 0 & 0 & \frac{X_3X_4}{X_1^2} & \frac{X_3X_5}{X_1^2} \\ -\frac{1}{X_1} & 0 & \frac{2X_3}{X_1^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{32}{X_1^2X_3^5} & 0 & -\frac{4}{X_1^2X_3^3} \\ 0 & \frac{X_3X_4}{X_1^2} & 0 & 0 & \frac{(4+X_3^4X_4^2)}{X_1^2X_3^3} & \frac{X_3X_4X5}{X_1^2} \\ 0 & \frac{X_3X_5}{X_1^2} & 0 & -\frac{4}{X_1^2X_3^3} & \frac{X_3X_4X_5}{X_1^2} & \frac{(3+2X_3^2X_5^2)}{2X_1^2X_3} \end{pmatrix} \tag{10}$$

and the vector element

$$[\mathbf{R}] = \sqrt{2\pi}\begin{pmatrix} -p_i\left(\frac{X_1}{X_3} + X_1X_3X_5^2 + \frac{X_1X_3^3X_4^2}{4}\right) + q_i\frac{X_1^3X_3}{\sqrt{2}} \\ +c_i\frac{2X_1^5X_3}{\sqrt{12}} - \gamma_r X_1X3 \\ -p_r\left(\frac{3X_1^2X_5}{X_3} + X_1^2X_3X_5^3 + \frac{3}{4}X_1^2X_4^2X_3^3X_5\right) + c_r\frac{2X_1^6X_3X_5}{\sqrt{12}} \\ +q_r\frac{X_1^4X_3X_5}{\sqrt{2}} + \gamma_i X_1^2X_3X_5 \\ -p_i\left(\frac{X_1^2X_5^2}{2} - \frac{X_1^2}{2X_3^2} + \frac{3X_1^2X_3^2X_4^2}{8}\right) - p_rX_1^2X_4 \\ +q_i\frac{X_1^4}{4\sqrt{2}} + c_i\frac{\sqrt{3}X_1^6}{18} - \gamma_r\frac{X_1^2}{2} \\ -p_i\frac{X_1^2X_4X_3^3}{4} - p_r\left(\frac{X_1^2X_3}{8} - \frac{X_1^2X_3^3X_5^2}{8} - \frac{3X_1^2X_4^2X_3^5}{32}\right) \\ -q_r\frac{X_1^4X_3^3}{16\sqrt{2}} - c_r\frac{\sqrt{3}X_1^6X_3^3}{72} - \gamma_i\frac{X_1^2X_3^3}{8} \\ -p_iX_1^2X_3X_5 + p_r\frac{X_1^2X_3^3X_4X_5}{2} \\ p_r\left(\frac{X_1^2}{X_3} + X_1^2X_3X_5^2 + \frac{X_1^2X_4^2X_3^3}{4}\right) - q_rX_1^4X_3 \\ -c_r\frac{2X_1^6X_3}{\sqrt{12}} - \gamma_iX_1^2X_3 \end{pmatrix} \tag{11}$$

which leads to the following explicit analytical expressions for the CVs equations of motion:

$$\dot{X}_1 = \gamma_r X_1 - p_r X_1 X_4 + p_i\left(2\frac{X_1}{X_3^2} + X_1X_5^2\right) - \frac{5\sqrt{2}}{8}q_iX_1^3 - \frac{4\sqrt{3}}{9}c_iX_1^5 \tag{12}$$

$$\dot{X}_2 = 2p_rX_5 + p_iX_3^2X_4X_5 \tag{13}$$

$$\dot{X}_3 = 2p_rX_3X4 + p_i\left(\frac{1}{2}X_3^3X_4^2 - \frac{2}{X_3}\right) + \frac{\sqrt{2}}{4}q_iX_1^2X_3 + \frac{2\sqrt{3}}{9}c_iX_1^4X_3 \tag{14}$$

$$\dot{X}_4 = 2p_r\left(\frac{4}{X_3^4} - X_4^2\right) + 8p_i\frac{X_4}{X_3^2} - \sqrt{2}q_r\frac{X_1^2}{X_3^2} - \frac{8\sqrt{3}}{9}c_r\frac{X_1^4}{X_3^2} \tag{15}$$

$$\dot{X}_5 = p_i\left(X_3^2X_4^2X_5 + 4\frac{X_5}{X_3^2}\right) \tag{16}$$

$$\dot{X}_6 = p_r\left(X_5^2 - \frac{2}{X_3^2}\right) + p_i\left(X_3^2X_4X_5^2 - X_4\right) + \frac{5\sqrt{2}}{8}q_rX_1^2 + \frac{4\sqrt{3}}{9}c_rX_1^4 + \gamma_i. \tag{17}$$

As the quintic CGLE is a generalized equation, it is interesting to remark that if the coefficients $p_i$, $q_i$, $c_r$, $c_i$, $\gamma_r$ and $\gamma_i$ are set to zero, we obtain a purely conservative model called the standard NLSE with the dispersion management [24]. The corresponding equations (12)–(17) in this case are the same as those obtained by the means of variational principle in [24], indicating that there is no shifting of the temporal position ($X_2 = 0$) for an initial value $X_5^0 = 0$. This fact is the consequence of the existence of the symmetry property between the CVs in the Gaussian ansatz [29, 30]. One should note that, using a different ansatz like hyperbolic secant or raised-cosine, bare approximation and variational principle lead to two different sets of pulse parameter equations.

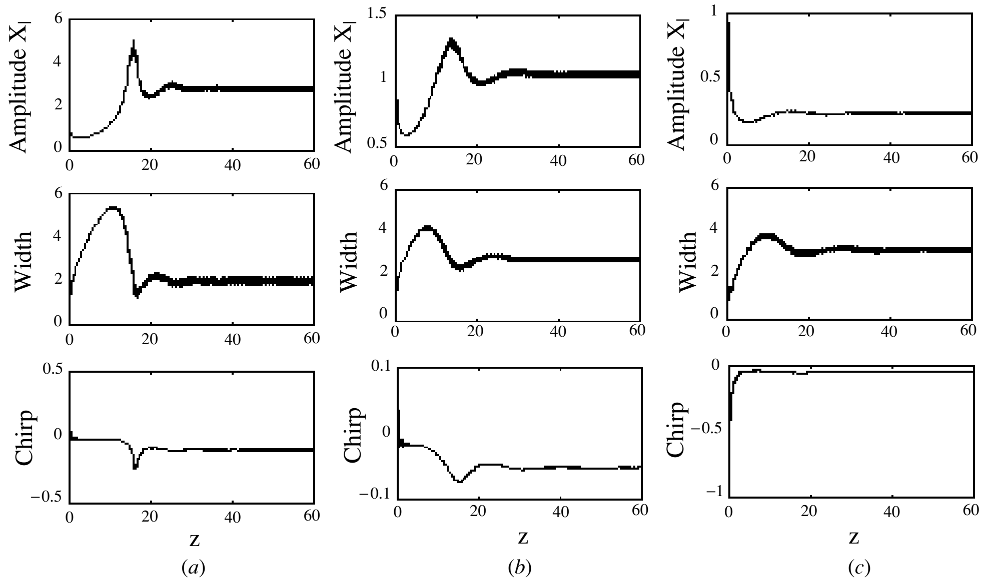



**Figure 1.** Evolution of the amplitude, pulse width and chirp versus the propagation distance for different values of the nonlinear coefficient $q_r$ ($q_r = a^2$), for the DM fibre line. (*a*) $a = 0.5$, (*b*) $a = 1$, (*c*) $a = 4$.

## 3. Numerical investigations

Using the standard fourth-order Runge–Kutta method, we carried out numerical studies of the evolution of the pulse parameters along the propagation distance $z$ and also the effects of the nonlinear coefficient $q_r(z)$ and that of the higher-order correction term of the nonlinear refractive index. It allows us to integrate the system of equations obtained by the CV analysis, with the spatial step taken to be 0.02 and with the following sets of fixed initial conditions $(X_1, X_2, X_3, X_4, X_5, X_6) = (1, 0, 1, 0, 0, 0)$. In this perspective, we considered parameters of equation (1) as follows:
$p = p_r(z) - i0.6$; $q = a^2 - i4.6 * 10^{-2}$, with $q_r(z) = a^2$; $c = c_r(z) + i1.6 * 10^{-3}$ and $\gamma = 0.16$. The dispersion profile that we consider with zero-average dispersion consists of an anomalous-dispersion ($p_r = d_1 > 0$) fibre with a length $z_1 = 0.2$, followed by a normal-dispersion ($p_r = d_2 < 0$) fibre of length $z_2 = 0.05$. Hence, a portion of DM fibre has a length $z_a = z_1 + z_2$. Nonlinear gain element and filters are chosen with a gain function
$G = 1 + q_i z_a \mid \psi \mid^2 + c_i z_a \mid \psi \mid^4$ and with a transfer function $T(\omega) = \exp[(\gamma_r + p_i\omega^2)z_a]$, respectively.

The great number of independent parameters of equation (1) makes the study of the whole-dimensional parameter space very complicated. So, we mainly focused on the DM system under consideration and we vary nonlinear coefficients in order to observe and analyse the evolution of characteristic parameters of the pulse propagating inside the fibre, and consequently the evolution of the pulse. At first, one should note that the main important or fundamental and well-known result is that any attempt of numerical resolution of CVs equations of motion (see equations (12–17)) leads to a form of solution only in the case of zero-average dispersion $\left(\sum_{i=1}^{2} d_i z_i \approx 0\right)$. In the following, numerical computations will be done for zero-average dispersion setting $d_1 = 1$ and the parameter $d_2$ remains, in all the

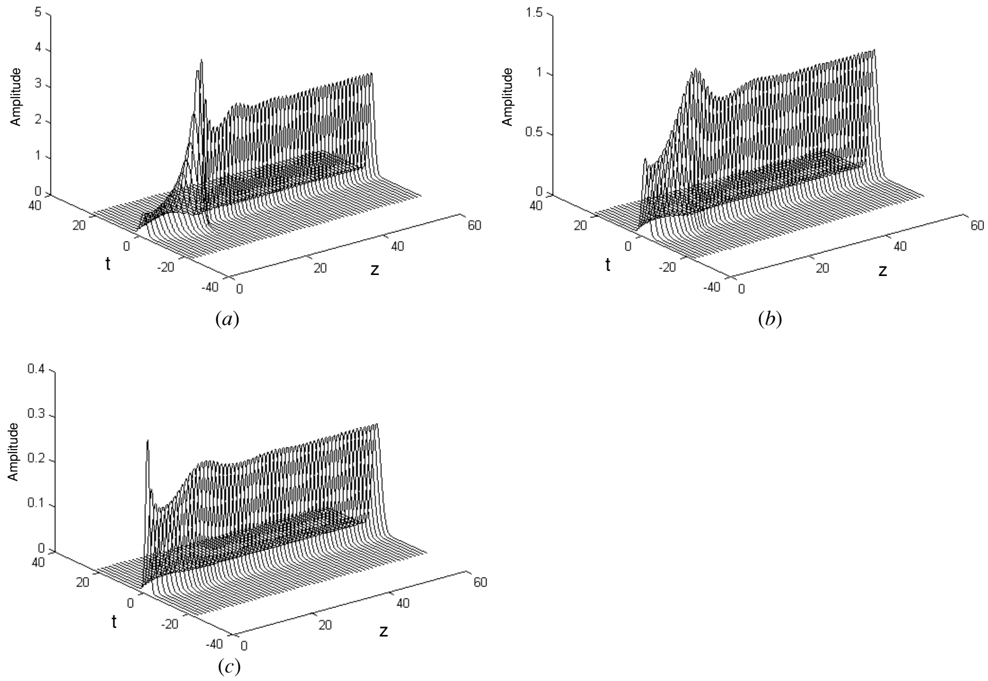



**Figure 2.** Waveforms of dispersion-managed soliton along the propagation distance. The parameters are the same as in figure 1. (*a*) $a = 0.5$, (*b*) $a = 1$, (*c*) $a = 4$.

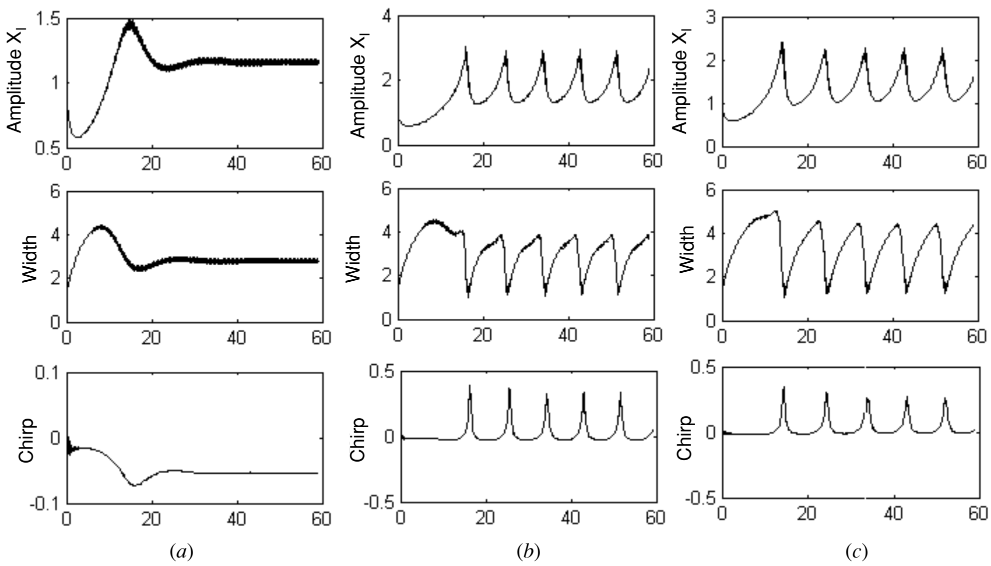



**Figure 3.** Evolution of the amplitude, pulse width and chirp versus the propagation distance for different values of the higher-order correction term of nonlinear refractive index $c_r$ ($a = 1$), for the DM fibre line. (*a*) $c_r = -0.1$, (*b*) $c_r = -0.32$. (*c*) $c_r = -0.6$.

text, $-4d_1$. Taking into account the nonlinear coefficient term, we focused our attention on three different values of this dimensionless parameter, say $a = 0.5$, $a = 1$ and $a = 4$. This

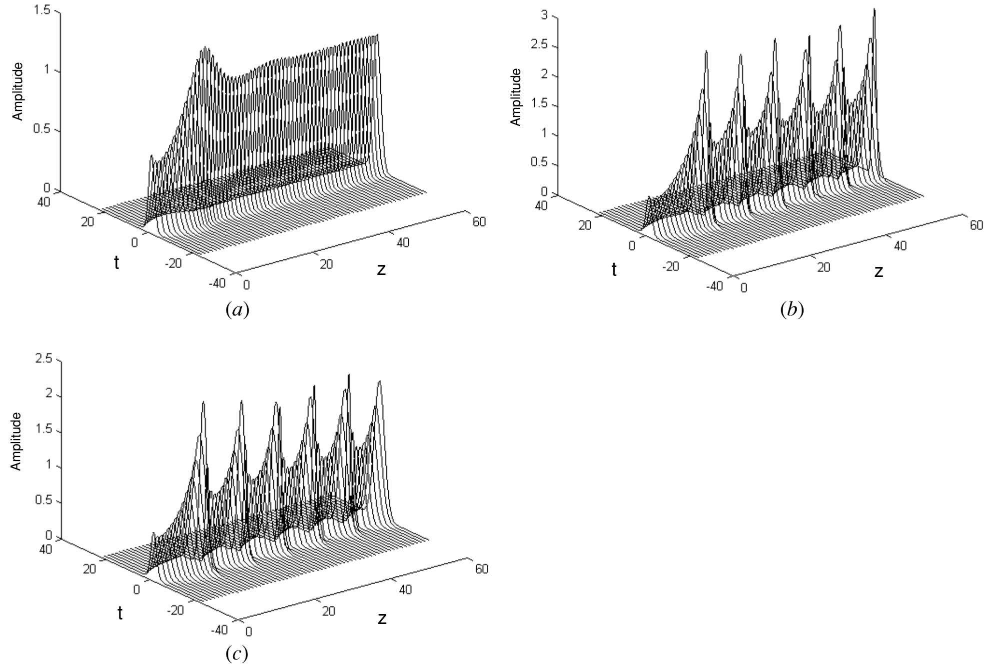



**Figure 4.** Waveforms of dispersion-managed soliton along the propagation distance. The parameters are the same as in figure 3. (*a*) $c_r = -0.1$, (*b*) $c_r = -0.32$, (*c*) $c_r = -0.6$.

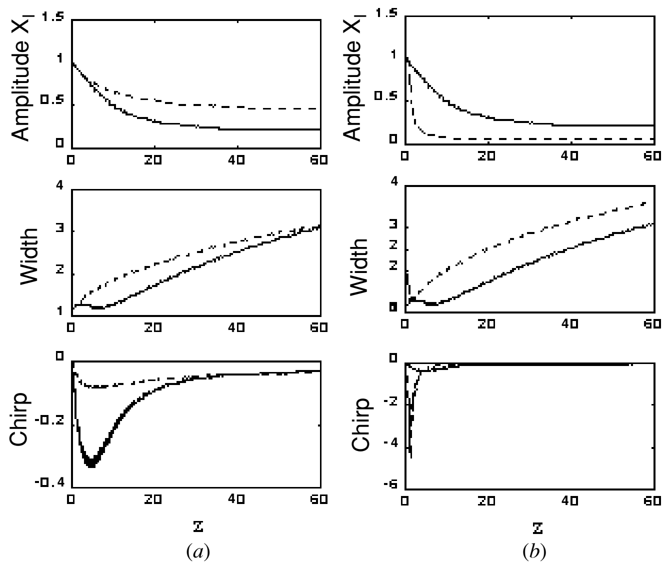
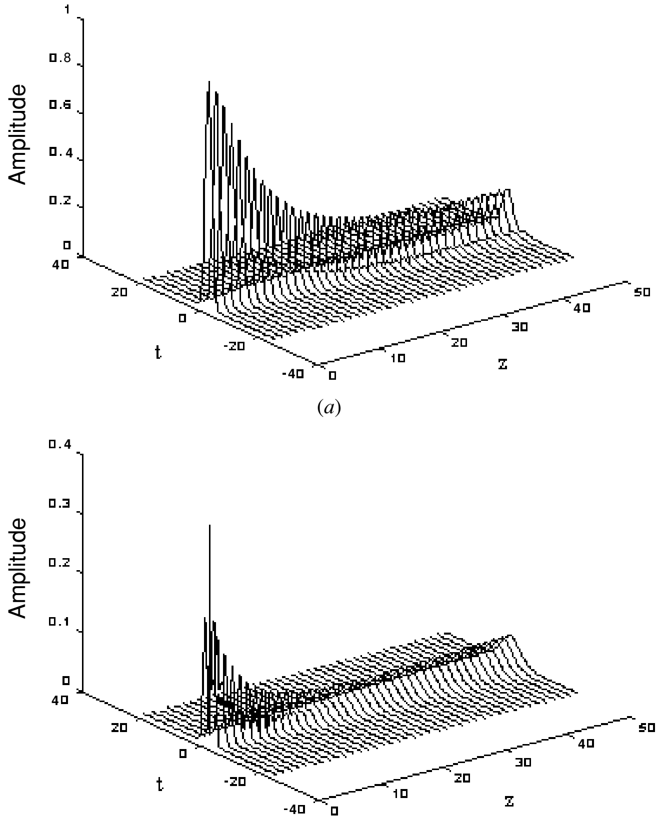



**Figure 5.** Evolution of the amplitude, pulse width and chirp for three values of the nonlinear refractive index when guiding filters and nonlinear gain are located after each period $z_a$ of DM fibre line. (*a*) $a = 0.5$ (dotted lines) and $a = 1$ (solid lines), (*b*) $a = 4$ (dotted lines) and $a = 1$ (solid lines).

choice is dictated by the fact that the parameter $a$ is related to the soliton order and governs the relative importance of the self-phase modulation and group-velocity dispersion effects




(*a*)

(*b*)

**Figure 6.** Different propagation results of the Gaussian initial pulse along the propagation distance for two values of the nonlinear refractive index. (*a*) $a = 1$, (*b*) $a = 4$.

on the pulse evolution along the fibre [1]. One observes, as shown in figure 1, a region of instability characterized by a great variation of the amplitude and pulse width. But, as the nonlinear coefficient increases, the fluctuations attenuate and the amplitude in the steady state decreases and becomes practically constant along the propagation distance. In order to better characterize the evolution of the pulse along the fibre, figure 2 shows waveforms of DM soliton for $a = 0.5$, $a = 1$ and $a = 4$. It is clearly seen that the amplitude and the pulse width reach stable values depending on the parameter $a$ after a distance of propagation. Consequently, the pulse propagation is stable in the given distance. Assuming the nonlinear coefficient $a = 1$, we basically pay attention to the saturation of the nonlinear refractive index. We then change slightly this parameter, and results of numerical simulations are presented in figure 3 for $c_r = -0.1$, $c_r = -0.32$ and $c_r = -0.6$. Figure 4 shows the evolution of the pulse with the same parameters as described in figure 3. We note a large variation of both amplitude and width along the propagation distance. In addition to the fluctuations which still exist, the quintic nonlinear term introduces a breathing mode inside the variation of both parameters, and this helps to stabilize the evolution of the pulse inside the fibre.

Secondly, we consider a more realistic case. Amplifiers, filters and nonlinear gain elements are located after each $d_2$ span and remain spaced by interval $z_a$. Coefficients $\gamma_r$, $p_i$, $q_i$ and $c_i$ are then expressed in terms of delta functions as we consider periodic-lumped

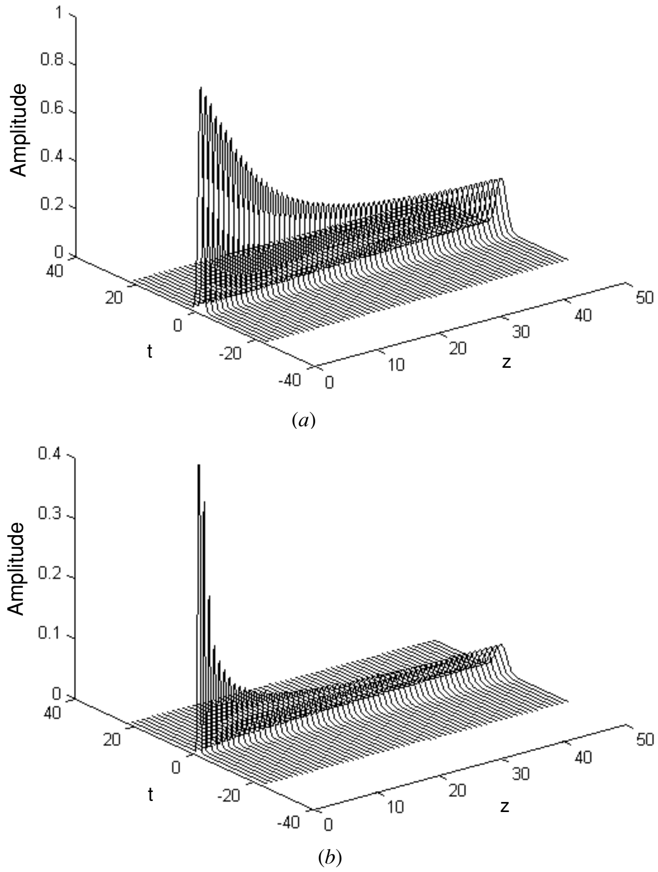



**Figure 7.** Waveforms of dispersion-managed soliton along the propagation distance obtained from numerical simulations of CVs equations for two values of the nonlinear refractive index. (*a*) $a = 1$, (*b*) $a = 4$.

filters and nonlinear gain elements. Note that whenever the guiding filter is located at the end of the normal-dispersion section, an instability may result and it may not be possible to suppress the pulse fluctuations without nonlinear gain [18]. In the rest of this paper, the parameter $d_1$ is set to 0.05 and $d_2$ remains, in all the text, $-4d_1$. Taking into account the nonlinear coefficient term, we pay attention to three different values of this parameter, say $a = 0.5$, $a = 1$ and $a = 4$. The fourth-order Runge–Kutta algorithm which we use to integrate equation of motion of CVs along the propagation distance $z$ allows to obtain results presented in figure 5 for the amplitude, the width and the chirp. Although the width variable $X_3$ does not present a breathing mode behaviour which is necessary to maintain the propagation of the non-conventional soliton, the amplitude first decreases and then reaches an equilibrium (steady state) where it remains practically constant. We note that the chirp converges practically from different values of the nonlinear coefficient to a unique value in the steady state. Once more, these curves indicate clearly that a stable pulse transmission can be achieved inside the DM fibre.

To confirm our CV analysis and the study of the pulse propagation inside the optical fibre, we performed a direct numerical simulations of the one-dimensional quintic CGLE (1) by using the split-step Fourier transform method with the time and space steps being 0.043 and

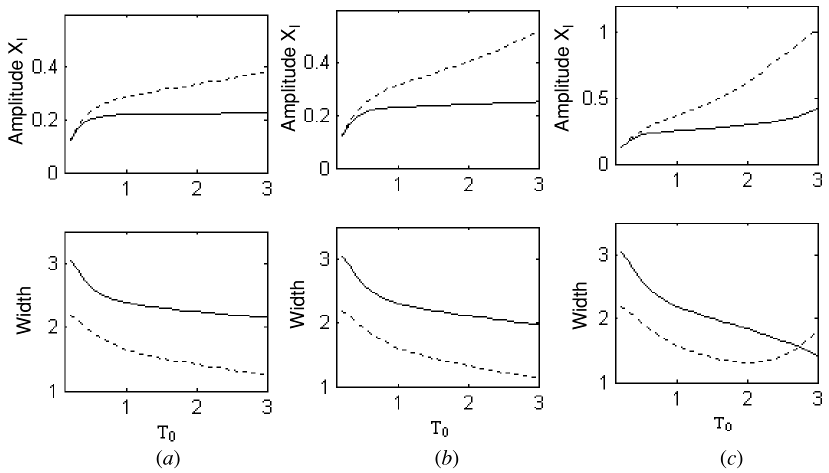



**Figure 8.** Variation of the amplitude and the width at $z = 100z_a$ (dotted lines) and $z = 200z_a$ (solid lines) as a function of the initial width when the nonlinear coefficient $a = 1$. (*a*) $c_r = -0.1$, (*b*) $c_r = -0.32$, (*c*) $c_r = -0.6$.

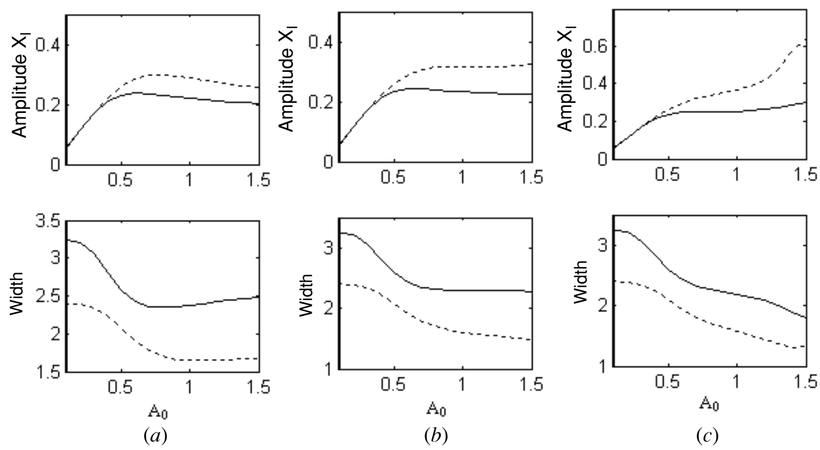



**Figure 9.** Variation of the amplitude and the width at $z = 100z_a$ (dotted lines) and $z = 200z_a$ (solid lines) as a function of the initial amplitude when the nonlinear coefficient $a = 1$. (*a*) $c_r = -0.1$, (*b*) $c_r = -0.32$, (*c*) $c_r = 0.6$.

0.025, respectively, and the initial Gaussian pulse $\psi(0, t) = \exp\left(-\frac{t^2}{2}\right)$. We recall that the parameters $p(z), q(z), c(z)$ and $\gamma(z)$ entered here are the same as those in the CV analysis. Figure 6 presents different propagation results of the Gaussian initial pulse for two values of the nonlinear refractive index. In order to compare the results obtained by the direct numerical resolution of equation (2) (described by figure 6) and those obtained by CVs equations, figure 7 presents waveforms of DM solitons obtained by CV analysis with the same parameters as in figure 6. Looking at those curves, we note a good agreement between both methods. This fact implies a good applicability of our CV results to the study of practical propagation of pulses along an optical fibre.

Our interest is now focused on the impact that initial parameters (basically initial amplitude and width) could have on the dynamic of CVs (basically the amplitude of the steady state).

Hirooka and Wabnitz [31] have shown that chirp, frequency and energy (that also means amplitude) converge from different initial values to a unique value in the steady state. As one can see in figure 5 for numerical integration of CVs equations or in figure 6 for the direct numerical integration of equation (1), the pulse reaches a constant amplitude after a certain distance inside the fibre. Does this amplitude which we suppose to be in a steady state remain unchanged if we vary the values of the parameter $q_r(z)$, $c_r(z)$ or the values of the initial pulse profile? Figures 8 and 9 present variation of the amplitude and the width at $z = 100z_a$ (dotted lines) and $z = 200z_a$ (solid lines) as a function of the initial width and the initial amplitude, respectively. A general view of those figures indicates that the amplitude and the width are not significantly modified when we slightly change the saturation of the nonlinear refractive index. One observes that the amplitude remains practically constant as the pulse evolves down the fibre (at $z = 200z_a$) depending on the initial width. Also, it is clear that there is a range of values of the initial amplitude which allows the amplitude to be practically constant. As far as the width is concerned, we note that its values become greater at $z = 200z_a$ than at $z = 100z_a$, depending on the initial width or the initial amplitude. This can be understood as the pulse broadens as it evolves down the optical fibre. These investigations allow to choose appropriate values of the initial parameters (e.g., 1 or 1.2 for the initial amplitude) of the initial Gaussian pulse shape launched into the DM fibre, and therefore, optimize the propagation of pulses.

## 4. Conclusion

In this work, we have successfully derived the CVs equations of motion for the quintic CGLE as perturbations of the NLSE with the help of the CV treatment of DM solitons including the residual field. The dynamics of pulse parameters can be deeply modified due to the effect of several terms in the quintic CGLE. We have noted a stabilizing effect of the nonlinear refractive index on the evolution of the amplitude and the pulse width. Also, it appears that the amplitude and the pulse width reach steady state depending on the initial value of the amplitude or the width. Results obtained by varying these initial values can permit us to choose the best Gaussian profile for the initial pulse and then optimize the propagation. In addition, we have mentioned in view of some propagation results presented in figures that the analytical results obtained by CV treatment are directly applicable to the study of the pulse propagation inside the optical fibre.

Finally, a recall of an interesting effect here is the chirped soliton stabilities indicated by Agrawal [32] and Manousakis *et al* [33]. Agrawal showed that the chirped solitons are stable only in the normal dispersion regime. In the case of anormalous dispersion, as is the case for erbium-doped fibre amplifiers, the amplified pulse develops many subpulses. However, the chirped solitons propagate undistorted only as long as the chirp verifies a restriction equation. This impractical restriction has been overcome by Manousakis *et al* [33], who performed a perturbed variational approach using the Pereira–Stenflo ansatz type pulse [34] to obtain a dynamical system due to the four characteristic parameters of interest for the pulse, namely: the amplitude, the time width, the chirp and the phase. One can see that the dynamical system of the four characteristic parameters of the pulse can be solved in order to obtain the corresponding differential equation of each parameter, which are similar in form to those derived in the present manuscript.

Although the use of the CV method has been treated in the bare approximation (neglecting the residual field), we obtained more accurate results in the dynamic of the pulse which is described here by two other parameters (temporal position and frequency) in addition to the four mentioned above. We can observe the qualitative behaviour of each DM soliton parameter along the propagation distance, which provides a better view of the propagating pulse. Particularly, as can be seen in some presented figures, the chirp acquires steady state along the propagation distance. To end, the bare approximation can lead to large discrepancies with respect to the full numerical approach in predicting some fundamental parameters such as the interactions distance of DM solitons [35]. In forthcoming studies, highly accurate representation of DM solitons on the basis of the minimization of the residual field associated with Hermite–Gaussian ansatz functions [36, 37], will be of an important interest. These studies can be surely useful for improving the performance of the communication systems.

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